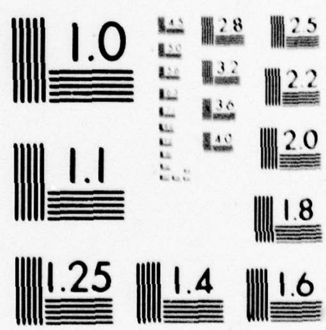
AD-A073 499 NAVAL UNDERSEA WARFARE CENTER SAN DIEGO CA F/G 20/1
AN EXTENSION OF THE THEORY OF ACOUSTIC TRANSMISSION THROUGHPORO--ETC(U)
NOV 67 P S CHASE, G L KINNISON

UNCLASSIFIED NUWC-TN-37 NL

| OF |

AD
A073499

END
DATE
FILMED
9-79
DDC


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

LEVEL II

(14) B1

MOST Project - 4

NUWC-TN-37

NAVAL UNDERSEA WARFARE CENTER

(11) Nove[redacted]967

(12) 36p.

(10) Palmer S. /Chase
Gerald L. /Kinnison

(6) AN EXTENSION OF THE THEORY OF ACOUSTIC TRANSMISSION THROUGH POROUS MATERIALS

by P.S. Chase & G.L. Kinnison

San Diego, California

Accession For	
NTIS GRA&I	☒
DDC TAB	☐
Unannounced	☐
Justification	
By Per Hr on file	
Distribution/	
Availability Codes	
Dist.	Avail and/or special
A	

SUBPROJECT NO. SR0010101

TASK NO. 0401

403 023

JUB

AN EXTENSION OF THE THEORY OF ACOUSTIC TRANSMISSION THROUGH POROUS MATERIALS

Palmer S. Chase and Gerald L. Kinnison

I. INTRODUCTION

On the subject of sound absorption by porous materials much work has been done but, unfortunately, before many recent engineering developments and apparatus. There now exists the opportunity to check some of the earlier results and theory derived for sound absorption in porous materials to determine whether the proposed models yield measured properties close enough to the theoretical values to be useful. L. L. Beranek and C. Zwikker both derived theoretical expressions for transmission of a single frequency through a porous media. The aim in this memorandum is to follow Beranek's derivation, which appeared in J. Acoust. Soc. Amer., July 1947, entitled Acoustical Properties of Homogeneous, Isotropic Rigid Tiles and Flexible Blankets, and included more mathematical steps for ease in following the derivation, and to extend some of his results in the treatment of soft blankets in anticipation of long range needs for their application. The information is published as a Technical Memorandum at this time to make the information available to a limited number of persons who have immediate need for it.

II. PARAMETERS USED IN THE DERIVATIONS

Most of the symbols used by Beranek, and in this article, represent well-known quantities, but there are a few whose appearance needs some explanation. Beranek follows Zwikker in the use of the structure factor k. Kosten stated that the value of k is generally greater than 3. According

to Kuhl and Meyer, $k\rho_0$ represents the total mass vibrating in one cubic centimeter, so that the partial motion of the skeleton is accounted for. Zwikker interprets k as the ratio of the volume of pores in the material to the volume of the pores that run straight through the material, which he then calls the lateral cavity factor. Figures 1 through 3 show the fiber randomness and structure in a typical fiberglass sample. Figures 2 and 3 are closeups of the sample in Figure 1 taken from the top and edge respectively. It is readily seen that the fibers are closer packed in the direction passing down in the sample (as viewed in Figure 1) than in either transverse direction. The structure factor, k, is an artificial constant to take up some slack between the theoretical and experimental evidence. It tries, in one lump, to take care of the effects of the presence of the skeleton on the acoustic wave passing through the material. It has been interpreted in many ways, one of which is the lateral cavity factor, the reason being that a more easily measured and hence verifiable concept is obtained. In general, k appears to try to account for the effect of the skeletal structure of the absorption process since little is known about this mechanism.

The viscous resistance R_1 is approximately equal to the resistance to a stream of constant velocity gas, called the specific flow resistance. R_2 is due to frictional forces among the fibers themselves and is normally considered negligible.

The coupling factor γ (which Beranek writes as γ_{12}) is introduced by Beranek as a result of the difference between the velocities of the air and the skeleton within the sample. It attempts to account for the apparent coupling between them which causes one to move when the other does.

Zwikker states that after extensive tests, he found that the porosity and the specific flow resistance do not provide sufficient information to determine the acoustic properties of a material. The structure factor and the coupling factor have been devised to better explain the acoustic properties.

III. LIST OF SYMBOLS

These have been taken directly from Beranek's article since the notation will be the same in this one.

a, b	Complex propagation constants in cm^{-1}.
α	Attenuation constant in nepers/cm. To convert from nepers to dB multiply the former by 8.686.
Θ	Phase angle
c	Velocity of sound in cm/sec.
d	Depth of sample of material in cm.
j	$(-1)^{1/2}$
k	Structure factor as used by Beranek and Zwikker. Dimensionless.
ℓ	Depth of a sample of material or column of air in cm.
K	Volume coefficient of elasticity of air where: $K = (\ell/\xi)(F_1/S_1)$ and ξ is an incremental displacement of a piston with area S_1 acting with a force F_1 against a column of air of length .
ω	Angular frequency = $2\pi f$
p	Sound pressure in $dyne/cm^2$ as measured by a microphone.
p_2	Average excess pressure in $dyne/cm^2$ exerted by a sound wave against the matter contained in the material, F_2/S.
q_1	Vector velocity of air particles in cm/sec.
q_2	Vector velocity of the gross movement of the solid particles caused by the influence of a sound wave in cm/sec.

Q	Volume coefficient of elasticity of an acoustical material where $Q = (\ell/\xi_2)(F_2/S)$ dyne/cm^2; and ξ_2 is an incremental displacement of a piston with area S acting with a force F_2 against a column of material contained in a vented cylinder of length ℓ.
R	Unit area acoustic resistance in rayls; also the specific flow resistance of a cubic centimeter of material.
R_1	Alternating specific viscous resistance offered per unit volume of air because of a difference between q_1 and q_2. The units are rayl/cm.
R_2	Alternating frictional resistance offered per unit volume of the solid matter because of friction among the fibers themselves, that is, due to a gradient in q_2.
Rayl	Unit of unit area acoustic impedance, dyne-sec/cm^3. And the unit of specific acoustic impedance.
ρ_o	Density of air in g/cm^3.
ρ_2	Density of the solid matter in the acoustic material, g/cm^3.
ρ_m	Density of the acoustical material in g/cm^3. Approximately, $\rho_m = \rho_2(1 - Y)$.
S	Area of a section of material = $S_1 + S_2$.
S_1	Area of air in cm^2.
S_2	Area of solid matter in cm^2.
t	Time in seconds.
τ	Coupling factor per unit volume of the air in the material. Introduced because of a difference between q_1 and q_2. $\tau = R_1 + j\omega(k-1)\rho_o$ Beranek uses τ_{12} for this symbol.
u	Component of the vector velocity q in the x direction.
V	Infinitesimal volume of acoustical material, $V = V_1 + V_2$.
V_1	Infinitesimal volume of air in cm^3.
V_2	Infinitesimal volume of solid material in cm^3.
ξ	Incremental displacement of the particles of air or solid material.
Y	Porosity, equals the ratio of the volume of voids to the total volume of a sample. $Y = V_1/(V_1 + V_2)$.

Z	Unit area acoustic impedance in rayls. $Z = p/q$.
Z_i	Characteristic impedance of an acoustical material in rayls. $i = 0, 1, 2, 3$.
Z_T	Initial impedance of a sample of material; i.e., at $x = 0$.

IV. SOME FORMULAS WORTH NOTING

1. $\tau = R_1 + j\omega(k-1)$
2. $Y = V_1/(V_1 + V_2)$
3. $Z = p/u\big|_{x=d}$
4. $K = (\ell/\xi_1)(F_1/S_1)$
5. $Q = (\ell/\xi_2)(F_2/S)$
6. $\omega = 2\pi f$
7. $Z_1 = R_1 + j\omega\rho_o k$
8. $Z_2 = Z_2' + R_2(1-Y)\frac{\partial}{\partial x}$
9. $Z_2' = R_1 Y + j\omega[\rho_m + (k-1)$

V. DERIVATION OF A WAVE EQUATION

In the mathematical treatment of a wave propagating in a fibrous material it is expedient that we make some simplifying assumptions. First, it will be assumed that a large number of fibers run randomly through each infinitesimal volume of the material and that statistically an equal number of them run through each face of this volume element. An actual observation of a typical fiber shows that the natural construction by compression techniques stratifies the absorber and increases the number of fibers running laterally through a volume element. A one-dimension analysis will circumvent this discrepancy since we will then require only a constant number of fibers to pass through any given

unit-cross-sectional area. This requirement can be closely met. When acoustical waves impinge on the material there will be a compression wave propagated in the air in the spaces between the fibers. Also, a certain fraction of the fibers will be compressed into the volume element. That is, the skeleton will react much like a driven spring. This spring effect will vary with direction since the material is in general not isotropic and hence div q will be direction dependent. Here also a one-dimension analysis will remove this difficulty by reducing div q to $\frac{\partial q}{\partial x}$, and hence considering the material as propagating waves in one dimension only.

From considerations of continuity of mass we get two equations. Their difference arises because the gas is assumed to be compressible in Equation (1) while in Eq. (2) it is assumed that the compressibility of the individual fibers is negligibly small.

$$\text{(1)} \quad \text{div}\, q_1 + \frac{1}{\rho_0}\frac{\partial \rho_0}{\partial t} + \frac{1}{V_1}\frac{\partial V_1}{\partial t} = 0$$

$$\text{(2)} \quad \text{div}\, q_2 + \frac{1}{V_2}\frac{\partial V_2}{\partial t} = 0$$

It can be shown by taking an incremental volume element and compressing it slightly that $\frac{d\ell}{\ell} = \frac{d\rho_0}{\rho_0}$ which becomes, when we let $d\ell = \xi$, $\xi/\ell = d\rho_0/\rho_0$. Substituting the volume coefficients of elasticity, Q and K and the above relationship into Eqs. (1) and (2) we find that they reduce to:

$$\text{(3)} \quad \text{div}\, q_1 + \frac{1}{K}\frac{\partial p}{\partial t} - \frac{(1-Y)}{QY}\frac{\partial p_2}{\partial t} = 0$$

$$\text{(4)} \quad \text{div}\, q_2 + \frac{1}{Q}\frac{\partial p_2}{\partial t} = 0$$

Beranek remarks that K will be found to assume values between 10^6 and 1.4×10^6 dyne/cm^2 as the compressions vary from isothermal to adiabatic. Q will depend on the particular fiber chosen, but will fall in the region of 2000 to 10,000 dyne/cm^2 for most materials.

The equations of motion can be derived from the physical principle that says basically that the pressure change is proportional to the velocity of the mass in motion, and opposite in sign. Beranek writes the equations of motion in the following form.

(5)
$$-\frac{\partial p_2}{\partial x} = Z_2' q_2 - Y\gamma q_1 + (1-Y) R_2 \frac{\partial q_2}{\partial x}$$
$$= Z_2 q_2 - \gamma Y q_1$$

(6)
$$-\frac{\partial p_2}{\partial x} = Z_1 Y q_1 - \gamma Y q_2$$

where Z_1 and Z_2 are given by

$$Z_1 = R_1 + j\omega \rho_0 k$$
$$Z_2' = R_1 Y + j\omega[\rho_m + (k-1)\rho_0 Y]$$
$$Z_2 = Z_2' + (1-Y) R_2 \frac{\partial}{\partial x}$$

We note the existence of the $(1-Y)R_2\frac{\partial}{\partial x}$ term in Eq. (5) which is present to account for dissipation forces due to relative movement of the fibers with one another. If this frictional force is small, as we shall consider later, then $R_2 \sim 0$ and $Z_2 = Z_2'$. Since we are interested only in the steady state solution, we can assume it to be periodic and make the gimmick substitution $\frac{\partial}{\partial t} = j\omega$. Also, let D be the operator $\frac{\partial}{\partial x}$ and let div q have only a component in the x-direction. We can summarize the manipulations so far in this derived set of equations.

(7) $$Du_1 + \frac{1}{K} j\omega p - \frac{(1-Y)}{QY} j\omega p_2 = 0$$

(8) $$Du_2 + \frac{1}{Q} j\omega p_2 = 0$$

(9) $$-Dp = Z_1 Y u_1 - \gamma Y u_2$$

(10) $$-Dp_2 = Z_2 u_2 - \gamma Y u_1$$

This is a system of four equations in four unknowns and their first derivatives. To solve for p, first differentiate Eq. (9) with respect to x and then substitute for Du_1 and Du_2 from Eqs. (7) and (8). We then get

(11) $$-D^2 p = \frac{j\omega p_2}{Q}\left[Z_1(1-Y) + \gamma Y\right] - \frac{j\omega p}{Q} Z_1 Y$$

Differentiate Eq. (10)

(12) $$-D^2 p_2 = Z_2 Du_2 - \gamma Y Du_1$$

Eq. (12) is valid since $(DZ_2)u_2 = (Z_2 D)u_2 = Z_2 Du_2$ considering D and Z_2 both as differential operators. Again, substitute Eqs. (7) and (8) into Eq. (12) to obtain

(13) $$-D^2 p_2 = \frac{-j\omega p_2}{Q}\left[Z_2 + \gamma(1-Y)\right] + \frac{j\omega p}{K}\gamma Y$$

Solve Eq. (13) for $\frac{j\omega p_2}{Q}$ and substitute this into Eq. (11). Then

(14) $$-D^2 p = \frac{Z_1(1-Y) + \gamma Y}{Z_2 + \gamma(1-Y)}\left[D^2 p_2 + \frac{\gamma Y j\omega p}{K}\right] - Z_1 Y \frac{j\omega p}{K}$$

If now we differentiat Eq. (11) twice,

$$(15) \qquad -D^4 p = \left(Z_1(1-Y) + \gamma Y\right)\frac{j\omega}{Q} D^2 p_2 - \frac{Z_1 Y j\omega}{K} D^2 p$$

and solving Eq. (14) for $D^2 p_2$ we can substitute this expression into Eq. (15) to obtain finally:

$$(16) \qquad D^4 p - \frac{j\omega Z_2}{Q} D^2 p - \frac{j\omega Z_1 Y}{K} D^2 p - \frac{\omega^2 Y Z_1 Z_2}{KQ} p - \frac{j\omega\gamma(1-Y)}{Q} D^2 p + \frac{\omega^2 \gamma^2 Y^2}{KQ} p = 0$$

This now can be factored in the following way.

$$(17) \qquad \left\{\left[D^2 - \frac{j\omega Z_2}{Q}\right]\left[D^2 - \frac{j\omega Y Z_1}{K}\right] - \frac{j\omega\gamma(1-Y)}{Q} D^2 + \frac{\omega^2\gamma^2 Y^2}{KQ}\right\} p = 0$$

If here we assume R_2 to be negligible, implying the frictional dissipation among the fibers themselves is negligible, then $Z_2 = Z_2'$, a constant. Then we recognize Eq. (17) as being a homogeneous fourth order differential equation with constant coefficients. Beranek points out that the part that R_2 plays in the total absorption will have to be determined experimentally, implying that leaving it in the equations complicates them sufficiently to prohibit further derivation of useful solutions. From now on, we will understand Z_2 to be the constant Z_2'.

VI. SOLUTIONS TO THE WAVE EQUATION

If we treat Eq. (17) as a fourth order differential equation with constant coefficients it can be written as

$$\left\{[D^2-\beta][D^2-\gamma] - \delta D^2 + \eta\right\} P = 0 \qquad (18)$$

where the constants $\beta, \gamma, \delta, \eta$ can be seen to be the following from Eq. (17).

$$\beta = \frac{-j\omega Z_2}{Q} \qquad \delta = \frac{j\omega\tau(1-Y)}{Q}$$

$$\gamma = \frac{j\omega Y Z_1}{K} \qquad \eta = \frac{\omega^2\tau^2 Y^2}{KQ}$$

The roots to the associated equation are

$$\left.\begin{matrix} a^2 \\ b^2 \end{matrix}\right\} = \frac{\beta+\gamma+\delta}{2} \pm \frac{\sqrt{(\beta+\delta+\gamma)^2 - 4(\beta\gamma+\eta)}}{2} \qquad (19)$$

which implies that Eq. (17) has a solution of the following form, where A, B, C and E are arbitrary constants.

$$P = A e^{ax} + B e^{-ax} + C e^{bx} + E e^{-bx} \qquad (20)$$

From Eq. (19) we can write a and b to be:

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \left\{[\beta+\gamma+\delta]\left[\frac{1}{2} \pm \frac{1}{2}\left(1 - \frac{4(\beta\gamma+\eta)}{(\beta+\gamma+\delta)^2}\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}} \qquad (21)$$

If we now substitute the values of $\beta, \gamma, \delta, \eta$ the result is the following which can be written more compactly as Eq. (22).

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \left[j\omega\left(\frac{Z_2K + Z_1YQ + \tau K(1-Y)}{KQ}\right)\right]^{1/2}\left[\frac{1}{2} \pm \left[1 - \frac{4KQY(Z_1Z_2 - \tau^2 Y)}{[Z_2K + Z_1YQ + \tau K(1-Y)]^2}\right]^{1/2}\right]^{1/2} \qquad (22)$$

For a wave propagated in an infinite medium, A = C = 0, since the pressure must be bounded. In general, two waves will exist, corresponding to the two propagation constants a and b, each with a different velocity of propagation and a different rate of attenuation. One of these waves travels primarily in the skeletal material and the other travels through the gas in the pores of the material. With apparatus to test the propagation of an oscillatory wave through a material we will be able to test the assumption that two waves exist. Mathematically they exist, as has been shown. Also, these waves are not independent because of two coupling factors. One of them comes from the viscous properties of the gas and the fact that there is a frictional dissipative force existing as the gas passes over the fibers. This factor is what gives rise to the Υ in the equations derived. The second coupling factor arises because as the quantity of solid matter in an incremental volume of material increases, the volume available for gaseous occupation is reduced. This is mathematically expressed by the appearance of the $\frac{\partial g_2}{\partial t}$ term in Eq. (5). Since we are dealing with small amplitudes of particle motion, and hence volumetric change, it seems reasonable to expect this quantity to be negligible. The combined result of a small R_2 and small $\frac{\partial g_2}{\partial t}$ is that the term $(1-Y)R_2 \frac{\partial g_2}{\partial t}$ in Eq. (5) is neglected in the derivation of the wave equation. And hence $Z_2 \doteq Z_2'$.

Beranek remarks that equation (22) is difficult to use as written, and that in practice two types of materials can be treated readily due to simplifying assumptions in Eq. (22). These are soft acoustical blankets and hard acoustical tiles. Both types have wide practical use, and acoustical materials in the intermediate range are few, a justification for the special treatment of these two specific types.

VII. SOFT ACOUSTICAL BLANKETS

Beranek defines soft blankets as those for which

$$(23) \quad 4KQ(Z_1 Z_2 - \Upsilon^2 Y)Y \ll (Z_1 YQ + Z_2 K + (1-Y)\Upsilon K)^2$$

Values of R_1 and ρ_m that are frequently measured require that $K > 20Q$ for the above inequality to hold. Values of R, the specific flow resistance, ρ_m, Y, Q and K/Q measured on several acoustical materials are given in Table I and are copied direct from Beranek's report to give the reader some feeling for the magnitudes of these quantities. For the purpose of determining K/Q Beranek assumed a value of about 1.2×10^6 for K.

Table I

Material	ρ_m g/cm^2	Y	R g sec^{-1} cm^{-3}	Q	K/Q
Fiberglass AA	0.0112	0.996	58	2800	400
Fiberglass AA	0.0074	0.997	34		
Fiberglass H-33 (XM-PF)	0.0416	0.983	29	8000	150
J. M. Stonefelt	0.0541	0.969	28	10000	120
Type "M"	0.0426	0.97	31	9000	130
Kapok-cotton	0.0498	0.96	118	9000	130
Wood fiber	0.0322	-	39	4000	300

With the definition for soft blankets, Eq. (23) yields:

$$a \doteq \left[\frac{j\omega}{KQ}(Z_1 YQ + Z_2 K + (1-Y)\Upsilon K)\right]^{1/2}\left[\tfrac{1}{2}+\tfrac{1}{2}\right]^{1/2}$$

Which simplifies to

$$a \doteq \left[\frac{j\omega}{Q}\right]^{1/2}\left[\frac{Z_1 YQ}{K} + Z_2 + \Upsilon(1-Y)\right]^{1/2}$$

If we impose further that $Q/K \ll 1/20$, a becomes

$$(24) \quad a \doteq \left[\frac{j\omega}{Q}\right]^{1/2}\left[Z_2 + \Upsilon(1-Y)\right]^{1/2}$$

A similar approximation for b is facilitated by the following algebraic approximation.

$$\left(\frac{1}{2} - \frac{1}{2}(1-X)^{1/2}\right)^{1/2} \doteq \left(\frac{X}{4}\right)^{1/2}$$

Then b becomes

$$b \doteq \left[\frac{j\omega}{K}\left(Z_1 Y + \frac{Z_2 K}{Q} + \frac{\gamma K}{Q}(1-Y)\right)\right]^{1/2}\left[\frac{(KQY(Z_1Z_2 - \gamma^2 Y))^{1/2}}{Z_2 K + Z_1 YQ + \gamma K(1-Y)}\right]$$

And then imposing the inequality $K \gg 20Q$

(25) $$b \doteq \left[\frac{j\omega}{K}\right]^{1/2}\left[\frac{(Z_1 Z_2 - \gamma^2 Y)Y}{Z_2 + \gamma(1-Y)}\right]^{1/2}$$

And when $(\omega\rho_m)^2 \gg (\mathrm{Re}\ Z_2) = R_1 Y$ then

(26) $$b \doteq \left[\frac{j\omega}{K}\right]^{1/2}\left[(R_1 + j\omega\rho_0 k)Y\right]^{1/2}$$

And if we let $R_1 \doteq 0$, $k = 1$ and $Y = 1$, then Eqs. (24) and (25) reduce to

(27) $$a \doteq j\omega\left[\frac{\rho_m}{Q}\right]^{1/2}$$

(28) $$b \doteq j\omega\left[\frac{\rho_0}{K}\right]^{1/2}$$

Here a is the propagation constant for a wave traveling in the skeleton alone, and b for a wave in the gas alone.

VIII. RIGID OR DENSE ACOUSTICAL TILES

We follow Beranek in his definition of a rigid tile as one for which

(29) $$Z_1 Q > 50 Z_2 K$$

For tiles, the magnitude of Z_1 is close to R_1 and Z_2 is about $\rho_m\omega$, so that Q/K must be greater than $50\omega\rho_m/R_1$. This condition is met only for very hard tiles, for which a and b readily become:

(30) $$a \doteq [j\omega]^{1/2}\left[\frac{(R_1 + j\omega\rho_0 k)Y}{K}\right]^{1/2}$$

(31) $$b \doteq [j\omega]^{1/2}\left[\frac{(Z_1 Z_2 - \gamma^2 Y)}{Z_1 Q}\right]^{1/2}$$

If again we let $R_1 = 0$ and $k = 1$, then a becomes the propagation constant for a wave traveling in the gas, and b for a wave in the skeleton, so that the roles of a and b have changed places in going from a soft blanket to a rigid tile. When Eq. (23) holds, $K/Q > 20$ and the skeleton borne wave is rapidly attenuated and has low velocity. In a rigid tile, when Eq. (31) holds, the wave in the skeleton is not attenuated appreciably and travels with a high velocity. At some intermediate value of Q, both waves will travel with equal velocity. When attenuation of a narrow frequency band desired the thickness and Q can be adjusted so that the two waves will arrive at the second face out of phase and advantage can be taken of attenuation by cancellation.

We now turn our attention to finding p_2, u_1 and u_2. Eq. (2) gives p as

(20) $$p = Ae^{ax} + Be^{-ax} + Ce^{bx} + Ee^{-bx}$$

If we differentiate this twice and substitute into Eq. (11) we can solve for p_2. Then

(32) $$p_2 = -Q\left\{\left[Ae^{ax} + Be^{-ax}\right]\left[\frac{Ka^2 - Z_1 Y j\omega}{j\omega K[Z_1(1-Y) + \gamma Y]}\right] + \left[Ce^{bx} + Ee^{-bx}\right]\left[\frac{Kb^2 - Z_1 Y j\omega}{j\omega K[Z_1(1-Y) + \gamma Y]}\right]\right\}$$

To find u_1 we substitute p and p_2 into Eq. (7) to obtain Du_1 and then integrate once with respect to x to obtain u_1. The result is:

(33) $$u_1 = -\left\{\left[Ae^{ax} - Be^{-ax}\right]\left[\frac{Ka^2(1-Y) + j\omega(Z_1 - j\omega\rho_0)Y^2}{KYa(Z_1 - j\omega\rho_0 Y)}\right] + \left[Ce^{bx} - Ee^{-bx}\right]\left[\frac{Kb^2(1-Y) + j\omega(Z_1 - j\omega\rho_0)Y^2}{KYb(Z_1 - j\omega\rho_0 Y)}\right]\right\}$$

We can find u_2 by using Eq. (8) and integrating once with respect to x. The answer becomes:

$$(34) \quad u_2 = \left[Ae^{ax} - Be^{-ax}\right]\left[\frac{Ka^2 - j\omega Z_1 Y}{Ka(Z_1 - j\omega\rho_0 Y)}\right] + \left[Ce^{bx} - Ee^{-bx}\right]\left[\frac{Kb^2 - j\omega Z_1 Y}{Kb(Z_1 - j\omega\rho_0 Y)}\right]$$

IX. NORMAL ACOUSTIC IMPEDANCE

The normal acoustic impedance at x = d is defined by taking the ratio p/u, where u is the particle velocity just outside the sample at x = d and p is the acoustic pressure at this point. At x = d, $u = -(u_1 Y + u_2(1-Y))$. For the acoustic impedance of a flexible blanket we can use the fact that in a soft blanket the skeleton borne wave travels slowly and is attenuated highly, justifying $u_2 = 0$. Then $u = Yu_1$. Since the wave in the skeleton is attenuated greatly, the pressure and velocity are composed primarily of the components with propagation constant b. Then

$$p = Ce^{bx} + Ee^{-bx}$$

$$u = Y\left[Ce^{bx} - Ee^{-bx}\right]\left[\frac{Kb^2(1-Y) + j\omega Y^2(Z_1 - j\omega\rho_0)}{KYb(Z_1 - j\omega\rho_0 Y)}\right]$$

$$(35) \quad p/u = \frac{1}{Y}\left[\frac{Ce^{bx} + Ee^{-bx}}{Ce^{bx} - Ee^{-bx}}\right]\left[\frac{KYb(Z_1 - j\omega\rho_0 Y)}{Kb^2(1-Y) + j\omega Y^2(Z_1 - j\omega\rho_0)}\right]$$

$$= \left[\frac{Ce^{bx} + Ee^{-bx}}{Ce^{bx} - Ee^{-bx}}\right]H$$

where H has the value given in Eq. (35). By definition $p/u\big|_{x=0} = Z_T = \frac{C+E}{C-E}H$. If we let $E/C = e^{-2\psi}$ then we can write that

$$(36) \quad p/u\big|_{x=d} = H \operatorname{ctnh}(bd + \psi)$$

From the equation above that states $Z_T = \frac{C+E}{C-E}H$, we can solve for E/C.

Using this and the exponential definition of ctnh x, we see that

$$E/c = \frac{Z_T - H}{Z_T + H} = e^{-2\psi} \quad ; \quad \mathrm{ctnh}\ \psi = \frac{Z_T}{H}$$

Consequently the impedance is given by the relation

$$(37) \qquad p/u\big|_{x=d} = \frac{Kb(Z_1 - j\omega\rho_0 Y)}{Kb^2(1-Y) + j\omega(Z_1 - j\omega\rho_0)Y^2}\,\mathrm{ctnh}\,(bd+\psi)$$

$$\psi = \mathrm{ctnh}^{-1}\ \frac{Z_T[Kb^2(1-Y) + j\omega Y^2(Z_1 - j\omega\rho_0)]}{Kb(Z_1 - j\omega\rho_0 Y)}$$

We can similarly find the impedance at x = d of a rigid tile. If it is mounted so that it does not vibrate as a whole, $u_2 = 0$, then the acoustic impedance will be given by: $p/u\big|_{x=d} = \frac{p}{-Y u_1}\Big|_{x=d}$

$$(38) \qquad p/u\big|_{x=d} = \frac{Ka(Z_1 - j\omega\rho_0 Y)}{Ka^2(1-Y) + j\omega(Z_1 - j\omega\rho_0)Y^2}\,\mathrm{ctnh}(ad+\psi)$$

$$\psi = \mathrm{ctnh}^{-1}\ \frac{Z_T[Ka^2(1-Y) + j\omega Y^2(Z_1 - j\omega\rho_0)]}{Ka(Z_1 - j\omega\rho_0 Y)}$$

For a rigid tile we can approximate still further when $(b^2K)^2 << (a^2K)^2 << (\omega\gamma)^2$. We rationalize Eq. (38) and apply the inequalities to finally obtain:

$$(39) \qquad p/u\big|_{x=d} = \frac{-jKa}{\omega Y}\,\mathrm{ctnh}\ (ad+\psi)$$

For an infinitely thick blanket, i.e., as $d \to \infty$, ~~ctnh~~ $\mathrm{ctnh}\,\infty$ = 1 and using the above inequality, $(b^2K)^2 << (\omega\gamma)^2$ we get that

$$(40) \qquad p/u\big|_{\infty} = \frac{-jKb}{\omega Y} \equiv Z_0$$

Z_o is called the characteristic impedance of the material and is defined in Eq. (40). It was shown earlier that the propagation constant for homogeneous and isotropic materials can be determined from a number of

specific parameters, namely: K, Q, ρ_o, ρ_m, Y, R_1, R_2. For acoustic blankets we found that K/Q was greater than twenty, and one of the waves, with propagation constant x a, traveled with low velocity and was attenuated highly in traveling through the material. The other has propagation constant b given by Eq. (25). When this equation is rationalized and the values substituted for Z_1, Z_2 and Υ, we can write it in the following form, which Beranek does.

$$b = j\omega\left(\frac{\langle\rho_1\rangle Y}{K}\right)^{1/2}\left(1 - j\frac{\langle R_1\rangle}{\langle\rho_1\rangle\omega}\right)^{1/2} \tag{41}$$

where $\langle\rho_1\rangle$ and $\langle R_1\rangle$ are given by

$$\langle\rho_1\rangle = \rho_o k\left\{\frac{R_1^2/[\rho_m\omega(1+\rho_o(k-1)/\rho_m]^2[Y+\rho_m/k\rho_o]+1/[1+\rho_o(k-1)/\rho_m]}{1+R_1^2/[\rho_m\omega(1+\rho_o(k-1)/\rho_m]^2}\right\} \tag{42}$$

$$\langle R_1\rangle = R_1\left\{\frac{[1+\rho_o(kY-1)/\rho_m+\rho_o^2(k^2-1)Y/\rho_m^2][1+\rho_o(k-1)/\rho_m]^2}{1+R_1^2/[\rho_m\omega(1+\rho_o(k-1)/\rho_m]^2}\right\} \tag{43}$$

If, moreover, $\rho_o(k-1)/\rho_m \ll 1$ then $\langle\rho_1\rangle$ and $\langle R_1\rangle$ can be given by the approximations

$$\langle\rho_1\rangle = \rho_o k\left\{\frac{(R_1/\rho_m\omega)^2(Y+\rho_m/k\rho_o)+1}{1+(R_1/\rho_m\omega)^2}\right\} \tag{44}$$

$$\langle R_1\rangle = R_1/[1+(R_1/\rho_m\omega)^2] \tag{45}$$

Beranek refers to $\langle\rho_1\rangle$ as the effective density of the gas particles and to $\langle R_1\rangle$ as the dynamic or effective resistance. He notes that the

value of R_1 is nearly constant as a function of frequency and approximately equal to the specific flow resistance.

The value of K, the volume coefficient of elasticity of air will vary from about 1.0 to 1.4 x 10^6 dyne cm^2. This is evident from the definition of K and thermodynamic considerations.

$$K = \frac{\delta F}{\delta S} = \rho \frac{\Delta P}{\Delta \rho} = \frac{V \Delta P}{\Delta V} \doteq -V \frac{\partial P}{\partial V} \tag{46}$$

Isothermal action requires that PV = constant, so that Eq. 46 implies K = P. The minus sign appears because a positive change, increase in P, corresponds to a negative change, decrease in V. On the other hand, if the gas moves adiabatically then PV = constant. Consequently, $K = \gamma P$, where γ for air is about 1.4 near standard conditions. In general, the motion of the gas varies from isothermal at low frequencies to adiabatic for higher frequencies for many bases so that K will vary from 1.0 to 1.4 times the pressure as a function of frequency for air, and the result follows since P is nearly 10^6 dyn/cm^2 for normal atmospheric pressure.

$\langle \rho_1 \rangle$ approaches $(\rho_m + \rho_0 kY)$ at low frequencies since the mass reactance, $\omega \rho_m$, is so low that the fibers move with the gas. At high frequencies the fibers stand nearly immobile as the air flows past with effective density approaching $\rho_0 k$. Also, $\langle R_1 \rangle$ approaches R_1 for high frequencies when the fibers are stationary, and $(\omega^2 \rho_m^2 / R_1)$ for low frequencies where there is little relative motion and hence little viscous dissipation.

From Eq. (40) we can solve for K. By using an experiment to determine the characteristic imedpance, Z_0, we can experimentally determine the attenuation and phase constants of b. A theoretical prediction of b can be obtained by Eq. (25) and the two compared. This is precisely Beranek's

approach and shows graphically the change from isothermal to adiabatic as the frequency increases. This changeover occurs entirely within the 100 to 2000 cps range, and mostly between 100 and 1000 cps. Beranek's results indicated that R_1 was reasonably constant over a wide range of frequencies but that $\langle \rho_1 \rangle$ and $\langle R_1 \rangle$ were not. Assuming the structure factor k to be about 1.5 and the porosity greater than 0.95, Beranek asserts that the propagation constant for homogeneous and isotropic materials can be found with good accuracy.

X. FRICTIONAL RESISTANCE AMONG THE FIBERS

In solving Eq. (17) for p we assumed that $R_2 = 0$, so that $Z_2 = Z_2'$ and our calculations would be simplified. When R_2 is not zero then

$$Z_2 = Z_2' + R_2(1-Y)\frac{\partial}{\partial x}$$

and Eq. (17) would then be written

$$\left\{\left[D^2 - \frac{j\omega}{Q}(Z_2' + R_2(1-Y)D\right]\left[D^2 - \frac{j\omega Y}{K}Z_1\right] - D^2\left[\frac{j\omega\gamma(1-Y)}{Q}\right] + \frac{\omega^2\gamma^2 Y^2}{KQ}\right\}p = 0 \tag{47}$$

This is also a fourth order differential equation, but unlike Eq. (17), terms of the first and third orders are present. The auxiliary equation will in general have four distinct roots, none of them associates. Hence the solution can be interpreted to be four independent waves propagated through the material with its own characteristic velocity and attenuation constant.

When $R_2 \neq 0$ it means there exist frictional forces due to relative motion of the fibers among themselves which dissipate, or at least redistribute, energy and consequently alter the propagation of the wave as it progresses through the medium. Any energy lost is transformed

a priori to heating the gas in the interstices. In general, the effect would be to reduce the velocity and increase the attenuation in the skeletal borne wave, with the opposite effect on the wave propagated in the gas. The thermal effects would be greater on the gas than the material.

Hence R_2 acts as a thermodynamic coupling factor. In rigid tiles the skeleton remains immobile so R_2 is necessarily zero. In soft blankets at high enough frequencies so that the gas moves adiabatically, the skeleton will again have little motion relative to the gas and R_2 will again be zero. At very low frequencies the skeleton will move in phase with the gas and the system will act isothermally. With sufficient time for the system to equilibrate during each cycle, the energy lost from the skeleton to the gas would be returned and the effect would be the same as though R_2 were zero.

The determination as to whether R_2 is actually negligible is of considerable importance. We wish to determine its significance. Enclose a piece of absorptive material classed as a soft blanket into an evacuated chamber and fasten one face rigidly to a mechanism that will drive the surface at a known frequency. Then measure the motion of the opposite surface, perhaps using an optical technique. In effect, we will consider the problem of a plane wave impinging on one side of the material and then measure the velocity and attenuation as a function of thickness and frequency. We can then calculate the propagation constants for the skeleton alone, since the wave propagated in the gas will be absent. Thus we let $K = 0$. Also R_1 and ρ_0 will be zero which implies that Z_1 and Υ are also zero.

From Eq. (7), when $K = 0$, $p = 0$, and $D_p = 0$ in Eq. (9). Also, $\Upsilon = 0$ makes Eq. (10) become

$$(48) \qquad -Dp_2 = Z_2 q_2$$

Then differentiating Eq. (48) and substituting into Eq. (8) we find

$$(49) \qquad D^2 p_2 = D(Z_2 q_2) = Z_2 D q_2 = \frac{j\omega Z_2 p_2}{Q}$$

Or

$$(50) \qquad \left[D^2 - \frac{j\omega Z_2}{Q}\right] p_2 = 0$$

This becomes, upon substituting for Z_2

$$(51) \qquad \left[D^2 - \frac{j\omega}{Q} R_2(1-Y) D - \frac{j\omega Z_2'}{Q}\right] p_2 = 0$$

This equation seems reasonable since with no air present the pressure measured by a microphone, p, would be zero so the only pressure present would be that exerted by the material surface, p_2. Under the assumptions that R_1 and ρ_o equal zero, $Z_2' = j\omega\rho_m$ and Eq. (51) becomes

$$(52) \qquad \left[D^2 - \frac{j\omega}{Q} R_2(1-Y) D - \frac{\omega^2 \rho_m}{Q}\right] p_2 = 0$$

This equation has the following solution,

$$(53) \qquad p_2 = A e^{j\omega(a+b)x} + B e^{j\omega(a-b)x}$$

Where A and B are arbitrary constants and the constants a and b are given by

$$a = \frac{R_2(1-Y)}{2Q}$$

$$b = \sqrt{\frac{R_2^2(1-Y)^2}{4Q^2} + \frac{\rho_m}{Q}} = \sqrt{a^2 + \frac{\rho_m}{Q}}$$

The velocity in the gas, q_1, would be zero, and we need only solve for q_2. Using Eq. (8) we find that q_2 is:

$$(54) \qquad q_2 = \frac{A}{Q(a+b)} e^{j\omega(a+b)x} + \frac{B}{Q(a-b)} e^{j\omega(a-b)x} + C$$

If we use Eq. (10) to evaluate the constant C, we find that it must be zero unless a = 0. But this implies that $R_2 = 0$ and that the pressure is a pure undamped sine wave. Since this case is trivial we discard it and let C = 0.

The motional impedance of the material at a point x = d from the initial point where the wave enters the medium, is given by $p/u \Big|_{x = d}$.

Since $u = -(Yu_1 + (1-Y)u_2)$ and $u_1 = 0$, then $u = -(1-Y)u_2$. The impedance is then given by

$$(55) \qquad p/u = \frac{-1}{(1-Y)} \left[\frac{A e^{j\omega(a+b)x} + B e^{-j\omega(b-a)x}}{\frac{A}{Q(a+b)} e^{j\omega(a+b)x} - \frac{B}{Q(b-a)} e^{-j\omega(b-a)x}} \right]$$

With a little manipulation this expression becomes more useful.

$$(56) \qquad p/u = \frac{-Qa}{(1-Y)} - \frac{Qb}{(1-Y)} \left[\frac{A e^{j\omega bx} + \frac{b+a}{b-a} B e^{-j\omega bx}}{A e^{j\omega bx} - \frac{b+a}{b-a} B e^{-j\omega bx}} \right]$$

If we let $\frac{b+a}{b-a} \frac{B}{A} = e^{-2\psi}$ then the expression for the impedance can more simply be written

$$(57) \qquad p/u \Big|_{x=d} = \frac{-Q}{(1-Y)}\left[a + b\,\mathrm{ctnh}(j\omega bd + \psi)\right]$$

If the impedance at x = 0 is called the initial impedance and denoted by Z_T then from Eq. (56)

$$(58) \qquad Z_T = \frac{-Q}{(1-Y)}\left[a + b\left(\frac{1 + \frac{b+a}{b-a}\frac{B}{A}}{1 - \frac{b+a}{b-a}\frac{B}{A}}\right)\right]$$

Or that $\mathrm{ctnh}\ \psi = -\frac{1}{b}\left[\frac{(1-Y)Z_T}{Q} + a\right]$

In the case that $R_2 = 0$, we get the former result that $b = \sqrt{\frac{\rho_m}{Q}}$ and the derivation of the motional impedance yields

$$(59) \qquad p/u \Big|_{x=d} = \frac{-Qb}{(1-Y)}\,\mathrm{ctnh}\,(j\omega bd + \psi)$$

Where

$$(60) \qquad \psi = \mathrm{ctnh}^{-1}\left[\frac{-Z_T(1-Y)}{bQ}\right]$$

The measured values for the impedance can now be compared with these theoretical values to judge whether internal dissipative forces, R_2, are actually negligible.

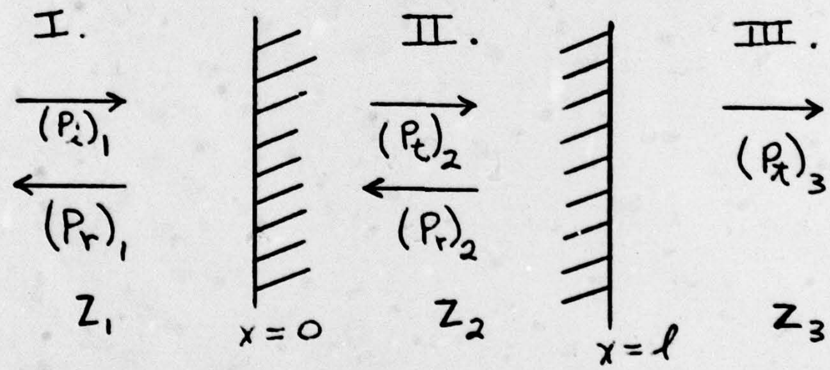
XI. ATTENUATION IN A TRANSMITTING MEDIUM

If we wish to consider attenuated, normally impingent, plane acoustic waves in a medium we can consider the velocity to be complex to account for the attenuation. This leads to the representation for pressure in the standard form $p = A\,e^{j(\omega t - \mathbb{K} x)}$ where $\mathbb{K}$ is the complex wavelength constant, $= \frac{\omega}{\mathbb{C}}$. Separating the real and imaginary parts by letting $\mathbb{K} = k - j\alpha$ then the expression

for p becomes

(61) $$p = A e^{-\alpha x} e^{j(\omega t - kx)}$$

The attenuation constant α is a lumped constant and may actually be a function of frequency or other parameters. We interpret its presence as due to viscous forces acting between the fibers and the air, or perhaps between the fibers themselves. The mechanism for α is not considered, but we assume that a representative value for a given material can be measured. The transmission coefficient α_T as a function of the mentioned measurable quantities is now sought. When $\alpha = 0$ we have only the acoustic impedance mismatch resulting from the acoustic wave propagation across a boundary as the mechanism for reduction in sound intensity. In this more general case we allow the second medium to absorb sound energy as well. General theory dictates that we set up the following model for three media with two interfaces. The first and third will be considered to be infinite.



P_i are incident waves, P_r are reflected waves and P_t are transmitted waves. The incident wave in medium I may be written $(P_i)_1 = A_1 e^{-\alpha_1 x} e^{j(\omega t - kx)}$ and the other waves may similarly be written:

(62) $$(P_r)_1 = B_1 e^{\alpha_1 x} e^{j(\omega t + k_1 x)}$$

(63) $$(P_t)_2 = A e^{-\alpha_2 x} e^{j(\omega t - k_2 x)}$$

$$(P_r)_2 = \mathbb{B}_2 e^{\alpha_2(x-\ell)} e^{j(\omega t + k_2 x)} \tag{64}$$

$$(P_t)_3 = \mathbb{A}_3 e^{-\alpha_3(x-\ell)} e^{j(\omega t - k_3(x-\ell))} \tag{65}$$

The constants $\mathbb{B}_1, \mathbb{A}_2, \mathbb{B}_2, \mathbb{A}_3$ may be complex quantities and physically accounting for a phase change at the interfaces. Also, $\alpha_1, \alpha_2, \alpha_3$ are the attenuation constants respectively for the three media. The Z_i are the respective characteristic impedances, given by the complex ratio of pressure to particle velocity, P_i/u_i.

The continuity of pressure at x = 0 gives rise to the equation

$$A_1 + \mathbb{B}_1 = \mathbb{A}_2 + \mathbb{B}_2 e^{-\alpha_2 \ell} \tag{67}$$

The continuity of particle velocity implies that

$$\frac{(P_i)_1}{Z_1} - \frac{(P_r)_1}{Z_1} = \frac{(P_t)_2}{Z_2} - \frac{(P_r)_2}{Z_2} \tag{68}$$

which becomes after substitution for the P_i:

$$Z_1(A_1 - \mathbb{B}_1) = Z_2(\mathbb{A}_2 - \mathbb{B}_2 e^{-\alpha_2 \ell}) \tag{69}$$

Similarly, continuity of pressure and particle velocity at x = l gives two more equations.

$$\mathbb{A}_2 e^{-\alpha_2 \ell} e^{-jk_2 \ell} + \mathbb{B}_2 e^{jk_2 \ell} = \mathbb{A}_3 \tag{70}$$

$$Z_3(\mathbb{A}_2 e^{-\alpha_2 \ell} e^{-jk_2 \ell} - \mathbb{B}_2 e^{jk_2 \ell}) = Z_2 \mathbb{A}_3 \tag{71}$$

The sound power transmission coefficient α_T is defined as the ratio of the intensity of the incident sound in medium I to the transmitted sound in medium III.

(72) $$\alpha_T = \frac{(P_t)_3^2 / 2Z_3}{(P_i)_1^2 / 2Z_1} = \frac{Z_1 \mathbb{A}_3^2}{Z_3 A_1^2}$$

Where $\mathbb{A}_3^2/A_1^2$ is the magnitude of the complex ratio of $\mathbb{A}_3$ and A_1. To find α_T we first can eliminate $\mathbb{B}_1$ from Equs. (67) and (69).

(73) $$A_1 = \frac{(Z_1+Z_2)\mathbb{A}_2 + (Z_2-Z_1)e^{-\alpha_2 \ell}\mathbb{B}_2}{2Z_2}$$

Eqs. (70) and (71) combine to give

(74) $$\mathbb{A}_3(Z_3 - Z_2) = 2Z_3\mathbb{B}_2$$

(75) $$\mathbb{A}_2 e^{-\alpha_2 \ell} e^{-jk_2 \ell} = \mathbb{A}_3\left(\frac{Z_3+Z_2}{2Z_3}\right)$$

Then the ratio $A_1/\mathbb{A}$ can be found from solving these equations. The result is:

(76) $$\frac{A_1}{\mathbb{A}_3} = \frac{(Z_1+Z_2)(Z_3+Z_2)e^{jk_2\ell}e^{\alpha_2\ell}}{4Z_2Z_3} + \frac{(Z_1-Z_2)(Z_2-Z_3)e^{-jk_2\ell}e^{-\alpha_2\ell}}{4Z_2Z_3}$$

Eq. (76) may be rewritten as

(77) $$\frac{A_1}{\mathbb{A}_3} = \frac{\cos k_2\ell}{4Z_2Z_3}\left[(Z_1+Z_2)(Z_2+Z_3)e^{\alpha_2\ell} + (Z_1-Z_2)(Z_2-Z_3)e^{-\alpha_2\ell}\right] + j\frac{\sin k_2\ell}{4Z_2Z_3}\left[(Z_1+Z_2)(Z_2+Z_3)e^{\alpha_2\ell} - (Z_1-Z_2)(Z_2-Z_3)e^{-\alpha_2\ell}\right]$$

With a little manipulation, and using the exponential definitions of sinh and cosh we are led to the final form for A_1/A_3.

$$(78) \quad \frac{A_1}{A_3} = \frac{[(Z_1^2-Z_2^2)(Z_2^2-Z_3^2)]^{1/2}}{2Z_2Z_3} \cos k_2 l \cosh(\alpha_2 l+\psi)$$

$$+j\frac{[(Z_1^2-Z_2^2)(Z_2^2-Z_3^2)]^{1/2}}{2Z_2Z_3} \sin k_2 l \sinh(\alpha_2 l+\vartheta)$$

Substituting the magnitude of this quantity into Eq. (72) we are led to the final result for α_T.

$$(79) \quad \alpha_T = \frac{4Z_1Z_3Z_2^2}{[(Z_1^2-Z_2^2)(Z_2^2-Z_3^2)]}\left[\cos^2 k_2 l \cosh^2(\alpha_2 l+\psi)\right.$$

$$\left.+\sin^2 k_2 l \sinh^2(\alpha_2 l+\vartheta)\right]$$

where

$$\psi = \cosh^{-1}\frac{Z_2(Z_1+Z_3)}{[(Z_1^2-Z_2^2)(Z_2^2-Z_3^2)]^{1/2}}$$

$$\vartheta = \sinh^{-1}\frac{Z_1Z_3+Z_2^2}{[(Z_1^2-Z_2^2)(Z_2^2-Z_3^2)]^{1/2}}$$

The theoretical characteristic impedance of a sample of material can be shown to be ρc, where ρ is the density of the material and c is the velocity of sound in the material, for a wave traveling in the positive direction. Letting $Z_i = \rho_i c_i$ for $i = 1, 2, 3$ and making the useful assumption that the characteristic impedances are related by the inequalities $\rho_1 c_1 \gg \rho_2 c_2 \gg \rho_3 c_3$

we obtain

$$(80) \qquad \alpha_T = \frac{4\rho_3 c_3}{\rho_1 c_1\left[\cos^2 k_2 l \cosh^2 \alpha_2 l + \sin^2 k_2 l \sinh^2(\alpha_2 l + \vartheta)\right]}$$

where $\psi = 0$

$$\vartheta = \sinh^{-1}\left(\frac{\rho_3 c_3}{\rho_2 c_2} + \frac{\rho_2 c_2}{\rho_1 c_1}\right) \simeq \frac{\rho_3 c_3}{\rho_2 c_2} + \frac{\rho_2 c_2}{\rho_1 c_1}$$

If, in addition, $\alpha_2 l$ is small, we can use the approximation that $\sinh^{-1} x \simeq x$ for small x to obtain the following results:

$$(81) \qquad \alpha_T = \frac{4\rho_3 c_3}{\rho_1 c_1\left[\cos^2 k_2 l + \sin^2 k_2 l \left(\alpha_2 l + \frac{\rho_3 c_3}{\rho_2 c_2} + \frac{\rho_2 c_2}{\rho_1 c_1}\right)^2\right]}$$

On the other hand, if $\alpha_2 l$ were large enough compared to ϑ so that the further inequality $e^{\alpha_2 l} \gg e^{-\alpha_2 l}$ held, then we could approximate α_T in this case by the expression

$$(82) \qquad \alpha_T = \frac{16\rho_3 c_3}{\rho_1 c_1} e^{-2\alpha_2 l}$$

A thick, or highly absorbing material, such as an acoustic blanket, rigidly attached to a stiff backing would satisfy the inequalities assumed in deriving the transmission coefficient given in Eq. (82).

Another useful approach is to let $\rho_1 c_1 = \rho_3 c_3 << \rho_2 c_2$ Then if we assume that $k_2 l$ is small enough that $\cos k_2 l \simeq 1$ and $\sin k_2 l \simeq k_2 l$, and going back to Eq. (77) we can derive an equivalent equation for this case.

$$(83) \qquad \alpha_T = \frac{4\rho_1^2 c_1^2}{\rho_2^2 c_2^2\left[\sinh^2\left(\alpha_2 l + \frac{2\rho_1 c_1}{\rho_2 c_2}\right) + (k_2 l)^2 \cosh^2 \alpha_2 l\right]}$$

For very small α_2 this expression becomes

$$\alpha_T = \frac{4\rho_1^2 c_1^2}{\rho_3^2 c_3^2 \left[\left(\alpha_2 l + \frac{2\rho_1 c_1}{\rho_2 c_2}\right)^2 + (k_2 l)^2 \right]} \tag{84}$$

When the second term in the denominator of the above expression is sufficiently larger than the first term that we can neglect the first term, we obtain the familiar mass law.

$$\alpha_T = \frac{4\rho_1^2 c_1^2}{\rho_2^2 c_2^2 (k_2 l)^2} \tag{85}$$

This might be more apparent if we replace $\rho_2 l$ by σ, the area density, $c_2 k_2$ by $2\pi f$, and then take the logarithm to the base ten of both sides of the equation to find the transmission loss, T.L., in dB. The recognizable result is:

$$\log \frac{1}{\alpha_T} = \text{T.L. (dB)} = 20 \log \frac{\pi}{\rho_1 c_1} + 20 \log \sigma f \tag{86}$$

If we let the medium be air, with a characteristic impedance of about 415 MKS rayls, and changing units so that σ is expressed in lb/ft^2 then Eq. (86) becomes

$$\text{T.L.} = -31.4 + 20 \log \sigma f \quad \text{(dB)} \tag{87}$$

We recall that this equation is valid for $k_2 l \ll 1$ and for all reasonable frequencies with the assumption that the second medium has characteristic impedance much greater than the surrounding medium.

When $\alpha_2 l$ is large in Eq. (84) it becomes

$$\alpha_T = \frac{16\rho_1^2 c_1^2 e^{-2\alpha_2 l}}{\rho_2^2 c_2^2 \left[1 + (k_2 l)^2\right]} \simeq \frac{16\rho_1^2 c_1^2 e^{-2\alpha_2 l}}{\rho_2^2 c_2^2} \tag{88}$$

which can also be written in terms of the Transmission Loss, in dB.

$$(89) \qquad T.L. = 20 \log \frac{4\rho_2 c_2}{\rho_1 c_1} + 0.868\, \alpha_2 l$$

Still another approach is to consider the loss due to medium II alone, and the interface between the second and third media. The problem then is reduced to finding the transmi~~tter~~ssion coefficient given by

$$(90) \qquad \alpha_T' = \frac{(P_t)_3^2 / 2Z_3}{(P_t)_2^2 / 2Z_2} = \frac{Z_2}{Z_3} \left| \frac{A_3}{A_2} \right|^2$$

For this case we can allow Z_2 to be complex.

From Eq. (75) this transmission coefficient is

$$(91) \qquad \alpha_T' = \left| \frac{4 Z_2 Z_3}{(Z_3 + Z_2)^2} \right| e^{-2\alpha_2 l}$$

Since the magnitude of Z_3 is much less than Z_2 by hypothesis then

$$(92) \qquad \alpha_T' = \frac{4\rho_3 c_3}{|Z_2|} e^{-2\alpha_2 l}$$

Allowing that the second medium is a soft blanket we can substitute for Z_2 the value of Z_o given in Eq. (40). The value of b as given in Eq. (26) is

$$(26) \qquad b = \left(\frac{j\omega}{K} \right)^{1/2} \left[Y(R_1 + j\omega\rho_0 k) \right]^{1/2}$$

from when the magnitude of Z_o can be computed and substituted into Eq. (92).

$$(93) \qquad \alpha_T' = \left(\frac{Y}{K} \right)^{1/2} \frac{4\rho_3 c_3 e^{-2\alpha_2 l}}{(\rho_0^2 k^2 + R_1^2/\omega^2)^{1/4}}$$

Analyzing Eq. (93) we see that the frequency dependence of α'_T decreases as ω increases. At lower frequencies the attenuation given by α'_T is more dependent on the frequency.

The import of these results is that knowing the transmission loss α_T of a sample of material of thickness ℓ, then we can find k as a function of the viscous absorption α_2. It appears that knowledge of the behavior of the absorption of an acoustical wave as it travels through the material, giving α_2, and a measure of the transmission loss after it has passed through one interface into air will yield enough information to determine k, the structure factor of the material. The assumptions we have made are briefly:

1. small amplitude of particle motion
2. $(b^2K)^2 \ll (a^2K)^2 \ll (\omega R_1)^2$
3. $Y > 0.95$
4. $Z_o \gg Z_3$

XII. APPLICATIONS

The theoretical results found in this memorandum can be directly applied to producing better sound transmission loss materials in the following manner. Sound propagation from a steel bulkhead through a sound-absorbing material and into air satisfies the model used for attenuation in a transmitting medium. As such, the theoretical results are applicable. Attenuation constants can be measured for various sound-absorbing materials and a prediction of the transmission loss obtained. Various aspects of the theory can then be readily checked. These are: (1) The equations for transmission loss, beginning with Eq. (80); (2)

By enclosing the sound-absorbing material in an evacuated chamber and optically measuring its displacement due to surface vibration, a check of the theory simplified for near zero atmospheric pressure can be made, as predicted in section X. Further, an estimate of the magnitude of R_2, the frictional resistance among the fibers themselves can be made. (3) For partial atmospheric pressures, known values of K and Q can be used to predict the values of the propagation constants a and b from which estimates of the variables k and γ can be made.

An attenuation tube, as originally designed by Scott, and a standing wave type impedance tube are now in use. Appropriation of materials and equipment for the previously suggested experiments has begun, including the acquisition of an optical tracker, vacuum equipment and transducer. The transducer is located in water and will drive the steel hull of a barge with the sample and optical tracking equipment in place on the inside of its hull. This method is simultaneously easy to set up in terms of available equipment and useful, since it will test samples in an actual operational environment.

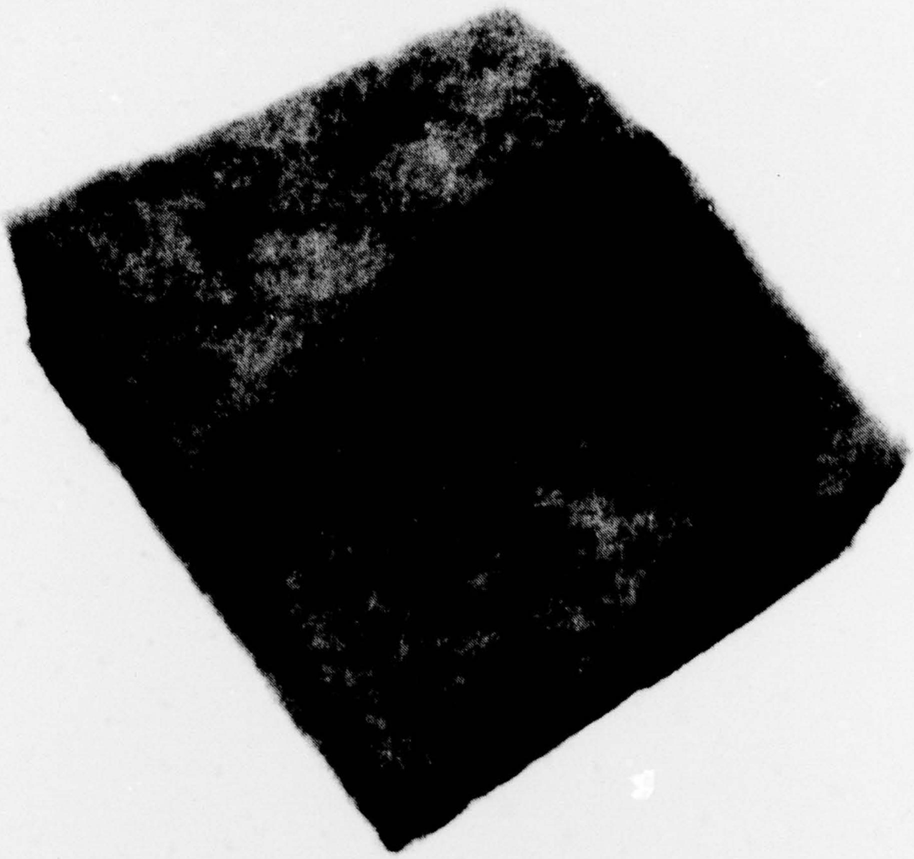


Fig. 1. Sample of fiberglass

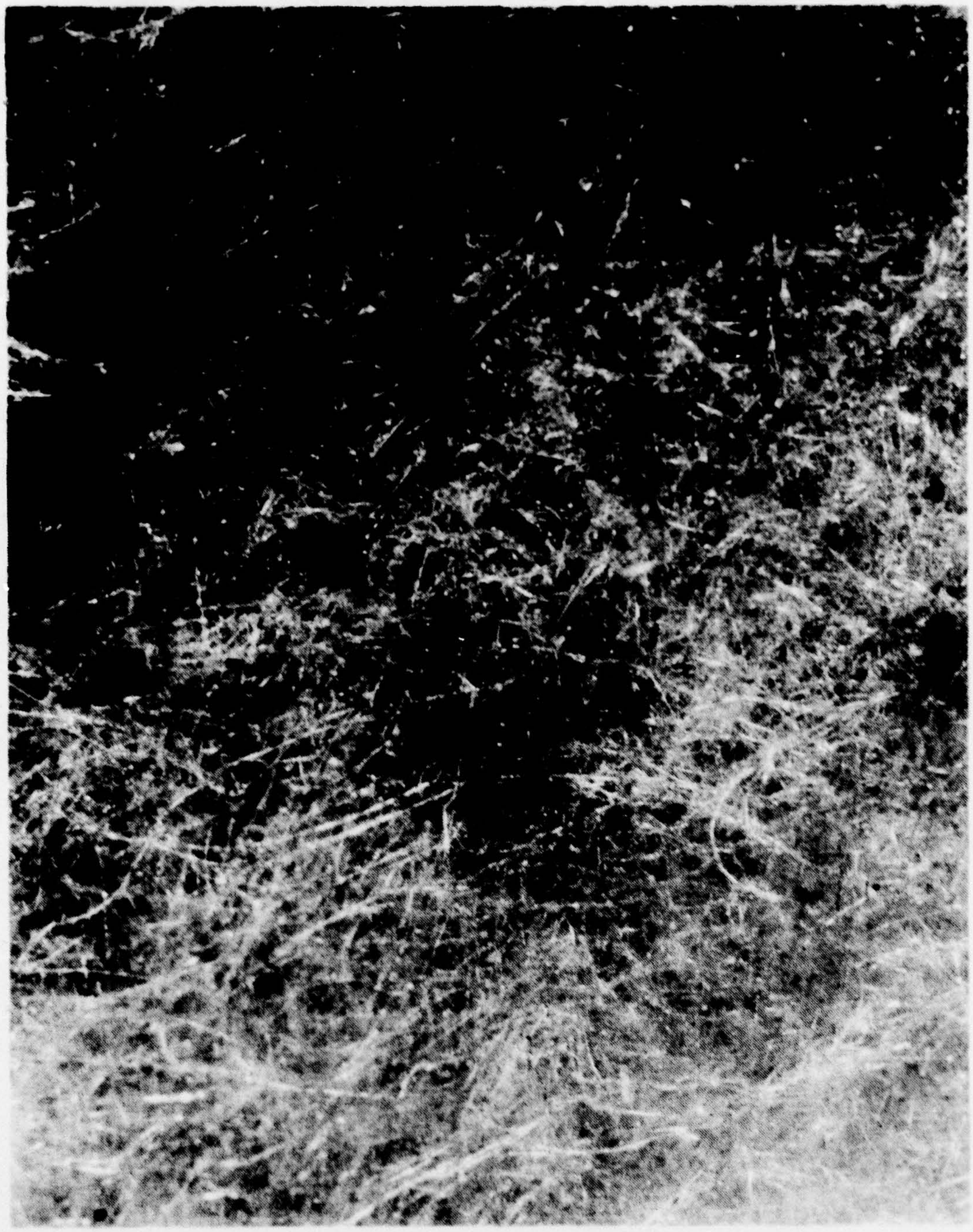


Fig. 2. Top view of sample

Fig. 3. Side view of sample